Name of College - S.S. College J. Bad
Deptt - Mathematics
Topic → Problem Based on Indeterminate Form (Cont'd) (Differential Calculus)
Class - B.Sc 2 (Hons)
Time - 10.15 A.M to 11 A.M
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[Problem based on the Form $\frac{\infty}{\infty}$]

Form $\frac{\infty}{\infty}$: $\lim_{x\to a} \frac{f(x)}{\phi(x)} = \lim_{x\to a} \frac{f'(x)}{\phi'(x)}$

1. Evaluate $\lim_{x\to a} \frac{\log(x-a)}{\log(e^x - e^a)}$

Since it is in the form $\frac{\infty}{\infty}$

Here $f(x) = \log(x-a) \Rightarrow f'(x) = \frac{1}{x-a}$

$\phi(x) = \log(e^x - e^a) \Rightarrow \phi'(x) = \frac{1}{e^x - e^a} \cdot e^x$

$$= \lim_{x\to a} \frac{\frac{1}{x-a}}{\frac{1}{e^x - e^a} \cdot e^x}$$

$$= \lim_{x\to a} \frac{e^x - e^a}{(x-a)e^x} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x\to a} \frac{e^x}{e^x + (x-a)e^x} \quad \text{(Applying L Hospital Rule.)}$$

$$= \frac{e^a}{e^a + (a-a)e^a} = \frac{e^a}{e^a} = 1$$

2. $\lim_{x\to 0} \frac{\log x^2}{\log \cot^2 x}$ it is of the form $\frac{\infty}{\infty}$

Here $f(x) = \log x^2 = 2\log x \Rightarrow f'(x) = \frac{2}{x}$

$\phi(x) = \log \cot^2 x = 2\log \cot x \Rightarrow \phi'(x) = \frac{2}{\cot x}(-\operatorname{cosec}^2 x)$

$$= -\frac{2\operatorname{cosec}^2 x}{\cot x}$$

$$= \lim_{x\to 0} \frac{2}{x\left[\frac{-2\operatorname{cosec}^2 x}{\cot x}\right]} = -\lim_{x\to 0} \frac{\cot x}{x \operatorname{cosec}^2 x}$$

$$= -\lim_{x\to 0} \frac{\cos x \sin x}{x} \quad \left[\frac{0}{0}\right]$$

Applying L-Hospital Rule

$$= \lim_{x\to 0} \frac{\cos^2 x - \sin^2 x}{1}$$

$$= \frac{1-0}{1} = 1$$

Evaluate $\lim_{x\to\infty} \frac{x^n}{e^x}$ $\left[\frac{\infty}{\infty}\right]$

$$= \lim_{x\to\infty} \frac{n x^{n-1}}{e^x} \quad \left[\frac{\infty}{\infty}\right]$$

$$= \lim_{x\to\infty} \frac{n(n-1) x^{n-2}}{e^x} \quad \left[\frac{\infty}{\infty}\right]$$

$$= \lim_{x\to\infty} \frac{n(n-1)(n-2) \cdots 3.2.1\, x^0}{e^x}$$

$$= \lim_{x\to\infty} \frac{\lfloor n}{e^x} = \frac{\lfloor n}{\infty} = 0 \quad \text{Since } \frac{1}{\infty} = 0$$

Problem based on $0 \times \infty$

$$\lim_{x\to\infty} g(x) \cdot f(x)$$

$$= \lim_{x\to\infty} \frac{g(x)}{\frac{1}{f(x)}} \quad \left[\frac{0}{0}\right]$$ Then Apply L-Hospital Rule.

$\lim_{x\to\infty} x \sin \frac{1}{x}$ $(\infty \times 0)$

Let $\frac{1}{x} = \theta$ $\quad x \to \infty \quad \theta \to 0$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{\theta\to 0} \frac{\cos\theta}{1} = \cos 0 = 1$$

$\lim_{x\to 0} (x \log x)$ Form $0\times\infty$

$= \lim_{x\to 0} \frac{\log x}{\frac{1}{x}}$

$= \lim_{x\to 0} \frac{1/x}{-1/x^2}$

$= \lim_{x\to 0} -\frac{x}{1} = 0$

Evaluate $\lim_{x\to 0} x \log \tan x$ it is of the form $0\times\infty$

$= \lim_{x\to 0} \frac{\log \tan x}{1/x} \quad \left(\frac{\infty}{\infty}\right)$

$= \lim_{x\to 0} \frac{\frac{1}{\tan x} \times \sec^2 x}{-1/x^2}$

$= -\lim_{x\to 0} \frac{x^2}{\sin x \cos x} \quad \left[\frac{0}{0}\right]$

$= -\lim_{x\to 0} \frac{2x}{\cos^2 x - \sin^2 x}$

$= -\lim_{x\to 0} \frac{2x}{\cos 2x} = \frac{-2\times 0}{1} = 0$

$\lim_{x\to\infty} 2^x \tan \frac{a}{2^x}$

it is of the form $0\times\infty$

$= \lim_{x\to\infty} 2^x \tan \frac{a}{2^x}$ Put $\frac{a}{2^x} = \theta$ when $x\to\infty$, $\theta\to 0$

$\Rightarrow 2^x = \frac{a}{\theta}$

$= \lim_{\theta\to 0} \frac{a}{\theta} . \tan\theta$

$[0/0]$

$= a \lim_{\theta\to 0} \frac{\tan\theta}{\theta} =$

$= a \lim_{\theta \to 0} \frac{\sec^2\theta}{1}$ Applying L-Hospital rule.

$= a \times 1 = a$.

Problems based on the form $\infty - \infty$

Method $\lim_{x \to a} [\phi(x) - \psi(x)]$ where $\lim_{x \to a} \phi(x) = \infty$, $\lim_{x \to a} \psi(x) = \infty$

$= \lim_{x \to a} \frac{\frac{1}{\psi(x)} - \frac{1}{\phi(x)}}{\frac{1}{\phi(x)\psi(x)}}$ Which is of the form $0/0$

{ then Apply L-Hospital Rule }

$\lim_{x \to \pi/2} [\sec x - \tan x]$ it is in the form $\infty - \infty$

$= \lim_{x \to \pi/2} \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

$= \lim_{x \to \pi/2} \frac{1 - \sin x}{\cos x}$ $\left[\frac{0}{0}\right]$

$= \lim_{x \to \pi/2} \frac{-\cos x}{-\sin x} = \frac{0}{1} = 0$

Problem :→ $\lim_{x \to \pi/2} [x \tan x - \pi/2 \sec x]$ it is in form $\infty - \infty$

$= \lim_{x \to \pi/2} \frac{x \sin x}{\cos x} - \frac{\pi}{2\cos x}$

$= \lim_{x \to \pi/2} \frac{2x \sin x - \pi}{2 \cos x}$ $[0/0]$

$$= \lim_{x \to \pi/2} \frac{\cancel{2}[\sin x + x\cos x - 0]}{-\cancel{2}\sin x}$$

$$= \frac{\sin \pi/2}{-\sin \pi/2} = -1$$

Find $\lim_{x \to 0} \frac{1}{x} - \cot x$ {It is in the form $\infty - \infty$}

$$= \lim_{x \to 0} \frac{\sin x - x\cos x}{x \sin x} \quad [0/0]$$

$$= \lim_{x \to 0} \frac{\cos x - [\cos x - x\sin x]}{\sin x + x\cos x}$$

$$= \lim_{x \to 0} \frac{x \sin x}{\sin x + x\cos x} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{\sin x + x\cos x}{\cos x + [\cos x - x\sin x]}$$

$$= \lim_{x \to 0} \frac{\sin x + x\cos x}{2\cos x - x\sin x} = \frac{0}{2} = 0$$

Evaluate

$$\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right]$$

it is in the form $\infty - \infty$

$$= \lim_{x \to 0} \frac{\sin^2 x - x^2}{x^2 \sin^2 x} \quad [0/0]$$

$$= \lim_{x \to 0} \frac{2\sin x \cos x - 2x}{x^2 \times 2\sin x \cos x + 2x \sin^2 x}$$

$$= \lim_{x \to 0} \frac{\sin 2x - 2x}{2x\sin^2 x + x^2 \sin 2x} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{2\cos 2x - 2}{2\left[\sin^2 x + x \times 2\sin x \cos x\right] + \left[2x\sin 2x + 2x^2\cos 2x\right]}$$

$$= \lim_{x \to 0} \frac{2\left[\cos 2x - 1\right]}{2\left[\sin^2 x + x\sin 2x\right] + 2\left(x\sin 2x + x^2\cos 2x\right)}$$

$$= \lim_{x \to 0} \frac{\cos 2x - 1}{\sin^2 x + x\sin 2x + x\sin 2x + x^2\cos 2x}$$

$$= \lim_{x \to 0} \frac{\cos 2x - 1}{\sin^2 x + 2x\sin 2x + x^2\cos 2x} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{-2\sin 2x - 0}{2\sin x\cos x + 2\left\{\sin 2x + 2x\cos 2x\right\} + \left\{2x\cos 2x - 2x^2\sin 2x\right\}}$$

$$= \lim_{x \to 0} \frac{-2\sin 2x}{3\sin 2x + 6x\cos 2x - 2x^2\sin 2x} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{-4\cos 2x}{6\cos 2x + 6\left[\cos 2x - 2x\sin 2x\right] - 2\left[2x\sin 2x + 2x^2\cos 2x\right]}$$

$$= \frac{-4 \times 1}{6 \times 1 + 6\left[1 - 0\right] - 2\left[0 + 0\right]} = -\frac{4}{12}$$

$$= -1/3$$